Decentralized Fair Resource Allocation for Relay-Assisted Cognitive Cellular Downlink Systems

Rui Wang, Vincent K. N. Lau, Cui Ying, Kaibin Huang, Bin Chen, and Xia Yang

***Abstract*—In this paper, we consider a relay-assisted cognitive cellular downlink system dynamically accessing a spectrum licensed to a primary network, thereby improving the efficiency of spectrum usage. A cluster-based relay-assisted architecture is proposed, where relay stations are employed for minimizing the interference to users in the primary network and achieving fairness for cell-edge users. Based on this architecture, an optimal solution is derived for jointly controlling data rate, transmission power, and subband allocation to optimize the weighted sum goodput where proportional fair scheduling (PFS) is included as a special case. As shown by simulations, the proposed solution achieves significant throughput gains and better user fairness compared with existing designs.**

I. INTRODUCTION

Dynamic spectrum access is a new paradigm to meet the challenge of the rapidly growing demands for broadband access and the spectrum scarcity for designing the next-generation wireless communication systems. A key obstacle for implementing dynamic spectrum access in cellular systems is that direct transmission from base stations to cell-edge users requires large power and thus causes strong interference to users in the primary network. As a result, the users in the cell-edge will have very small access opportunity due to the primary-user activity and this fairness issue cannot be solved simply by fair scheduling at the base station. Relay-assisted cellular system will be an effective solution for alleviating the above fairness issue because it helps to reduce the transmission power required to reach the mobiles on the edge. However, there are still a few critical issues associated with the design and operation of relay-assisted CR systems as summarized below.

- **Decentralized Power, Rate and Subband Allocation Algorithm**: Extensive research has been carried out on resource allocation in point-to-point relay-assisted communication systems. power and subband allocation for relay-assisted OFDMA systems is studied in [1]–[3]. However, these existing works considered centralized solution (e.g. at BS) and global knowledge of the system CSI (CSI between the mobiles, BS and RS) is required at the BS, which is very difficult to achieve in practice due to huge signalling overhead.
- **Fair Consideration**: Conventional relay-assisted cellular systems perform resource allocation to maximize the sum-throughput [1], [2]. Yet, fairness is an important requirement and general solution of fair scheduling in RS-assisted CR system is still not fully addressed.
- **Imperfect CSIT and Sensing Measurement**: In most of the existing works on resource allocation with relay-assisted systems [1], [2], or CR systems [4], [5], perfect CSIT and or perfect sensing measurement are usually assumed, which is again difficult to achieve in practice.

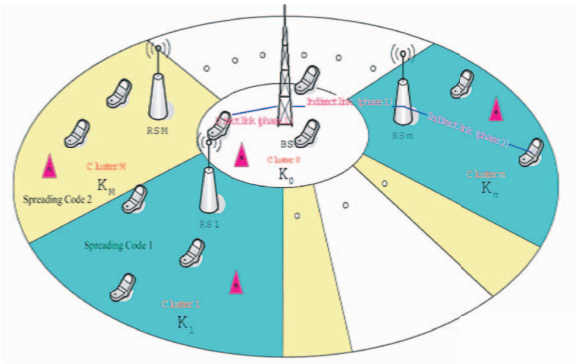

Fig. 1. Cluster based relay-assisted downlink cellular system.

In this paper, a cluster-based architecture is proposed for downlink relay-assisted cognitive cellular system. Based on the proposed architecture, we derive a low-complexity and decentralized algorithm for controlling power, rate, and subband allocation, which maximizes the weighted sum goodput (including proportional fair scheduling (PFS) as a special case) under the primary user interference constraint. The solution accounts for multiuser diversity, user fairness, and imperfect CSI and spectrum sensing. As shown by simulations, the resource allocation algorithm thus designed ensures scheduling fairness for cell-edge users.

The remainder of this paper is organized as follows. The system model is described in Section II. In Section III, the problem of optimally controlling power, rate, and subcarrier allocation is formulated; the solutions are presented in Section IV. Section V contains simulation results, followed by concluding remarks in Section VI.

II. SYSTEM MODEL

A. Architecture and Protocol

As illustrated in Figure 1, the single-cell cluster-based relay-assisted cognitive cellular downlink system consists of one base station (BS) transmitting to K mobile users (MS), where communications are assisted by M relay stations (RS). The cell is divided into $M + 1$ clusters as shown in Figure 1. The central cluster is indexed as the 0th cluster, where users

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directly communicate with the base station over relatively short distances. Each of the remaining M clusters is served by a half-duplexing RS. Specifically, each relay station forwards data packets from the base station to users in the corresponding cluster using the decode-and-forward (DaF) strategy. The number of users in the mth cluster is denoted as K_m.

The above secondary downlink system (SU) is assumed to opportunistically access a spectrum licensed to another network, where users are referred to as the *primary users* (PU) and have the highest priory of using the spectrum. Primary users are distributed over the service area of the SU system. To avoid interrupting the communication of primary users, every transmitter (including the BS and the relays) of the SU system is not allowed to transmit if there is an active PU in the coverage.

The channels are assumed to be frequency selective and divided into N independent subbands using the *orthogonal frequency division multiplexing* (OFDM) modulation. Downlink transmission is divided into frames. In phase one, the base station broadcasts to mobile users in the 0th cluster and all relay stations; in phase two, each relay station forwards data packets to users in the corresponding cluster on the subbands containing no primary users. To avoid interfering users in other clusters, every two neighboring relay stations transmit using two orthogonal spreading sequences. Interference of a relay station to users in non-adjacent clusters is assumed negligible due to path loss.

B. Channels

Channels are modelled as block fading. Thus the channel realization in one frame is assumed quasi-static and different realizations in different frames are assumed to be independent and identically distributed. Channel gains are characterized by path loss and microscopic fading. Based on this channel model, the symbol received at the kth user in the mth cluster over the nth subband, denoted as $Y_{m,n,k}$, can be written as

$$Y_{m,n,k} = \sqrt{p_{m,n,k} l_{m,k}} H_{m,n,k} X_{m,n,k} + Z_{m,n,k}, \qquad (1)$$

where $X_{m,n,k}$ is the transmitted symbol, $p_{m,n,k}$ the transmission power, $l_{m,k}$ the path loss, $H_{m,n,k} \sim \mathcal{CN}(0,1)$ models fading, and $Z_{m,n,k}$ is also $\mathcal{CN}(0,1)$ representing a sample of the additive white Gaussian noise process. Note that $H_{m,n,k}$ represents the channel between the kth user and the base station if $m = 0$, or the mth relay station if $m > 0$.

We consider a time division duplex (TDD) system where the CSIT estimation is not perfect due to the estimation noise and duplexing delay. Thus, the CSIT error model is given below:

$$\hat{H}_{m,n,k} = H_{m,n,k} + \Delta H_{m,n,k}, \quad \forall m, n, k \qquad (2)$$

where $H_{m,n,k}$ represents actual CSI, $\Delta H_{m,n,k}$ represents the CSIT error which is modeled as $\mathcal{CN}(0, \sigma_e^2)$, and $\mathbf{E}[\Delta H_{m,n,k} \hat{H}_{m,n,k}] = 0$. For convenience, transmit CSI is separated for different clusters and grouped as the sets $\hat{\mathbf{H}}_m = \cup_{n,k}\{\hat{H}_{m,n,k}\}$ for $0 \le m \le M$, which are referred to as *local CSIT*. The grand set $\hat{\mathbf{H}} = \{\hat{\mathbf{H}}_m\}$ is called the *global CSIT*.

C. Dynamic Spectrum Access

Each secondary user senses spectrum and searches for subbands unused by primary users, which, for instance, may be wireless microphones or other Part 74 devices. The spectrum sensing results consist of binary indicators specifying the availability of subbands, which are referred to as *raw sensing information* (RSI). Let $\hat{S}_{m,n,k} \in \{0,1\}$ denote the sensed state of kth user on the nth subband in the mth cluster, where $\hat{S}_{m,n,k} = 1$ and 0 correspond to the states "available" and "unavailable", respectively. RSI $\{\hat{S}_{m,n,k}\}$ is communicated by users to their corresponding servers (base/relay stations) for enabling resource allocation. Let $S_{m,n}$ be the actual primary-user state on the nth subband in the m cluster and $\mathbf{S} = \{S_{m,n}\}$ be the actual PU activity which is quasi-static over a number of frames[1]. Moreover, define $q_p = \Pr(S_{m,n} = 1)$, which is identical for all m and n. In practice, we cannot have perfect sensing at the mobile and there exist nonzero probabilities for the events *false alarm* $\left(q_f = Pr(\hat{S}_{m,n,k} = 0 | S_{m,n} = 1)\right)$ and *mis-detection* $\left(q_m = Pr(\hat{S}_{m,n,k} = 1 | S_{m,n} = 0)\right)$ [6]. Moreover, $q_d = 1 - q_m$ represents the probability of detection.

Due to the imperfect sensing measurement, it is not possible to eliminate the interference to the PU systems. To protect communication in the primary network, we require

$$I_{m,n} = p_{m,n} \tau_{m,n}^2 (1 - \mathbf{E}[S_{m,n} | \widehat{\mathbf{S}}_{m,n}]) \le I_0 \qquad (3)$$

where $I_{m,n}$ is the conditional average interference level (conditioned on the sensing measurements) from the SU (at the mth cluster and the nth subband) to the PU and $\widehat{\mathbf{S}}_{m,n} = \{\hat{S}_{m,n,k} | k \in \{1, K\}\}$.

III. JOINT CONTROL OF RATE, POWER AND SUBBAND ALLOCATION: PROBLEM FORMULATION

A. Definitions of Control Policies

The system resource allocation must satisfy the following constraints:

$$\sum_{n=1}^{N} \sum_{k=1}^{K_m} p_{m,n,k} \le P_m \quad \forall m, \qquad (4)$$

where P_m is the peak power constraint. By definition, the percentages of subband allocated to different users/relay-stations should satisfy

$$\sum_{n=1}^{N} \sum_{k=1}^{K_m} \alpha_{m,n,k} \le 1 \quad \forall m. \qquad (5)$$

Furthermore, the date rates are adjusted under a constraint on the per-link packet error probability[2] P_{out}, namely that for given $0 < \epsilon < 1$

$$P_{\text{out}}(\{r_{m,n,k}\}) = \epsilon \quad \forall m, n, k. \qquad (6)$$

We define the system resource allocation with respect to the system states as policies. The polices for controlling transmit power, subband sharing and transmit data rate at the

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[1] In practice, the PU activity changes over a longer time scale compared with the CSI.

[2] The PER is we considered is due to channel outage.

BS are defined as the function sets $\mathcal{P}_0 := \left\{p_{0,n,k}(\hat{\mathbf{H}},\hat{\mathbf{S}})\right\}$, $\mathcal{A}_0 = \left\{\alpha_{0,n,k}(\hat{\mathbf{H}},\hat{\mathbf{S}})\right\}$, and $\mathcal{R}_0 = \left\{r_{0,n,k}(\hat{\mathbf{H}},\hat{\mathbf{S}})\right\}$, which are adapted to the CSIT $\hat{\mathbf{H}}$ and RSI $\hat{\mathbf{S}}$. The policies used by a relay station depend on the packet receiving status of the phase one transmission. Let $t_{n,m} \in 0, 1$ denote the indicator of the decoding state of the mth relay station on the nth subband. Moreover, define the set $\mathbf{T}_m = \{t_{n,m}|\forall n\}$. Adding the newly defined sets as input, the policies for controlling power, rate, and subband allocation at relay stations are defined similarly to those for the base station as $\mathcal{P}_m := \left\{p_{m,n,k}(\hat{\mathbf{H}},\hat{\mathbf{S}},\mathbf{T}_m)\right\}$, $\mathcal{A}_m := \left\{\alpha_{m,n,k}(\hat{\mathbf{H}},\hat{\mathbf{S}},\mathbf{T}_m)\right\}$, and $\mathcal{R}_m = \left\{r_{m,n,k}(\hat{\mathbf{H}},\hat{\mathbf{S}},\mathbf{T}_m)\right\}$.

Each packet transmitting from the base station to a relay station is designed to contain information bits for users to be served by this RS in the cluster. Let $d_{m,n,k}$ be the fraction of kth user's information bits in a packet transmitted over the nth sub-channel and received at the mth relay station. It follows from the definition that

$$\sum_{k=1}^{K_m} d_{m,n,k} \leq 1, \ \forall m > 0, k, n. \tag{7}$$

The base station is assumed to control $\{d_{m,n,k}\}$ based on the CSIT and RSI. The corresponding control policy is defined as $\mathcal{D} := \left\{d_{m,n,k}(\hat{\mathbf{H}},\hat{\mathbf{S}})\right\}$.

B. Average Weighted Goodput

The average weighted goodput is defined and used in the sequel as the metric for optimizing control policies discussed in the preceding section. $C_{m,n,k}$ denotes the instantaneous mutual information between the mth transmitter and the kth receiver in the nth subband ((m,n,k)th subband), which is written as

$$C_{m,n,k} = g\alpha_{m,n,k}\log_2\left(1 + \frac{p_{m,n,k}l_{m,k}|H_{m,n,k}|^2}{\alpha_{m,n,k}}\right)$$

where g is equal to 0.5 for $m = 0$ and 0.25 for $m \geq 1$.[3] The instantaneous goodput over the the (m,n,k)th subband is defined as

$$U_{m,n,k} := S_{m,n}r_{m,n,k}\mathbf{I}(r_{m,n,k} \leq C_{m,n,k}). \tag{8}$$

where $\mathbf{I}(\cdot)$ is the indicator function.

Let $\{w_{m,k}\}$ represents a set of goodput weights for different users, whose values are set according to the users' QoS priorities. The average weighted goodput is given as system design:

$$\overline{G}(\mathcal{A},\mathcal{P},\mathcal{D}) = \mathbf{E}_{\hat{\mathbf{S}},\hat{\mathbf{H}}}\left\{\underbrace{\mathbf{E}_{\mathbf{S},\mathbf{H}}\left[\sum_{m,n,k} w_{m,k}U_{m,n,k} \mid \hat{\mathbf{S}},\hat{\mathbf{H}}\right]}_{\widetilde{G}(\mathbf{A},\mathbf{P},\mathbf{D}|\hat{\mathbf{S}},\hat{\mathbf{H}})}\right\}$$

where $\widetilde{G}$ defined above is the conditional average system goodput (conditioned on $\hat{\mathbf{S}},\hat{\mathbf{H}}$). To facilitate optimization, $\widetilde{G}$ is decomposed as

$$\begin{aligned}&\widetilde{G}(\hat{\mathbf{S}},\hat{\mathbf{H}})\\ =&\underbrace{\sum_{n=1}^{N}\sum_{k=M+1}^{K_0} w_{0,k}\beta_{0,n}r_{0,n,k}\big(1-\Pr[r_{0,n,k} > C_{0,n,k}|\hat{\mathbf{H}}]\big)}_{\widetilde{G}_0} + \sum_{m=1}^{M}\\ &\mathbf{E}_{\mathbf{T}_m}\left[\underbrace{\sum_{n=1}^{N}\sum_{k=1}^{K_m} w_{m,k}\beta_{m,n}r_{m,n,k}\big(1-\Pr[r_{m,n,k} > C_{m,n,k}|\hat{\mathbf{H}}]\big)}_{\widetilde{G}_m}\right],\end{aligned}$$

where $\beta_{m,n} = \mathbf{E}[S_{m,n}||\hat{\mathbf{S}}|]$ is the probability that the subband is available given the sensing feedbacks from the mobiles, and $\Pr[r_{m,n,k} > C_{m,n,k}|\hat{\mathbf{H}}] = 1 - \mathbf{E}_{\mathbf{H}}\left[\mathbf{I}(r_{m,n,k} \leq C_{m,n,k}) \mid \hat{\mathbf{H}}\right]$ is conditional packet error probability of one-hop link for given $\hat{\mathbf{H}}$ and $\hat{\mathbf{S}}$.

C. Problem Formulation

The policies defined in Section III-A are optimized for maximizing $\overline{G}(\mathcal{A},\mathcal{P},\mathcal{D})$ under the constraints (3), (4)-(6) and the flow balance constraint:

$$\sum_{n=1}^{N} r_{m,n,k} \leq \sum_{n=1}^{N} d_{m,n,k}t_{n,m}r_{0,n,m} \ \forall m > 0, k, t_{n,m}. \tag{9}$$

Since a policy consists of a set of actions for each realization of CSIT and RSI, finding the optimal policy is equivalent to the following problem.

Problem 1. *For each given CSIT $\hat{\mathbf{H}}$ and RSI $\hat{\mathbf{S}}$ realization, we have:*

$$\begin{aligned}\{\mathbf{A}^*,\mathbf{P}^*,\mathbf{D}^*\} = \max_{\mathbf{A}_0,\mathbf{P}_0,\mathbf{D}}\Bigg\{&\widetilde{G}_0(\mathbf{A}_0,\mathbf{P}_0|\hat{\mathbf{S}},\hat{\mathbf{H}}) + \sum_{m=1}^{M}\mathbf{E}_{\mathbf{T}_m}\\ &\left[\max_{\mathbf{A}_m,\mathbf{P}_m}\widetilde{G}_m(\mathbf{A}_m,\mathbf{D},\mathbf{P}_m|\hat{\mathbf{S}},\hat{\mathbf{H}},\mathbf{T}_m)\right]\Bigg\}\\ &\text{s.t. the constraints in (3), (4)-(6), and (9),}\end{aligned}$$

where $\mathbf{A}_m = \{\alpha_{m,n,k}|\forall m,n,k\}$, $\mathbf{P}_m = \{p_{m,n,k}|\forall m,n,k\}$, $\mathbf{D} = \{d_{m,n,k}|\forall m > 0, n, k\}$ *and* $\mathbf{A} = \{\mathbf{A}_m|\forall m\}$, $\mathbf{P} = \{\mathbf{P}_m|\forall m\}$.

Noting that neither the objective function nor the constraint (9) is convex, the traditional optimization approaches in [1], [2] cannot be applied in this problem. In the following, we shall show how to solve this non-convex optimization problem by decomposing the resource allocation between the BS and the RSs. Using primal-decomposition, we can separate Problem 1 into the following subproblems.

Sub-Problem 1 (Optimization at RS m).

$$\begin{aligned}&\widetilde{G}^*_m(\{r_{0,n,m}\},\{d_{m,n,k}\}|\hat{\mathbf{S}},\hat{\mathbf{H}},\mathbf{T}_m)\\ =&\max_{\mathbf{A}_m,\mathbf{P}_m}\sum_{n,k}\frac{(1-\epsilon)w_{m,k}\beta_{m,n}\alpha_{m,n,k}}{4}\log_2\Big(1+\frac{p_{m,n,k}}{\alpha_{m,n,k}}\varphi_{m,n,k}\Big)\end{aligned}$$

s.t. the constraints in (3), (4)-(6), *and* (9).

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[3] For $m = 0$, $g = 0.5$ accounts for throughput loss due to the base station being half-duplex; for $m > 1$, $g = 0.5$ combines the half-duplex loss at a relay station and the spreading factor of two needed for nulling the interference from the station to users in neighboring clusters.

where $\varphi_{m,n,k} = \frac{1}{2} l_{m,n} \sigma_e^2 F^{-1}_{|\hat{H}_{m,n,k}|^2/\frac{1}{2}\sigma_e^2}(\epsilon)$ [4], *which is obtained from the outage probability constraint.*

Sub-Problem 2 (Optimization at the BS)**.**

$$\begin{aligned} \max_{\mathbf{A}_0,\mathbf{P}_0,\mathbf{D}} \quad & \widetilde{G}_0(\mathbf{A}_0,\mathbf{P}_0|\hat{\mathbf{S}},\hat{\mathbf{H}}) + \sum_{m=1}^{M} \\ & \mathbf{E}_{\mathbf{T}_m} \widetilde{G}^*_m(\{r_{0,n,m}\},\{d_{m,n,k}\}|\hat{\mathbf{S}},\hat{\mathbf{H}},\mathbf{T}_m) \\ s.t. \quad & \textit{the constraints in } (3), \textit{ and } (4)\text{-}(6). \end{aligned}$$

IV. JOINT CONTROL OF RATE, POWER AND SUBBAND ALLOCATION: SOLUTIONS

A. Asymptotically Optimal Algorithm

Solution of Sub-problem 1: The Sub-problem 1 can be solved by using the standard duality approach [7], hence, we omit the details here due to the page limitations [5]. Note that the Sub-problem 1 is a non-convex optimization problem because the optimization constraint is non-convex. Nevertheless, since the problem satisfies the property of "time sharing" as introduced in [8], the dual gap of the above problem is zero, and hence, solving the above dual problem will lead to the optimal solution of Sub-problem 1.

Solution of Sub-problem 2: Observe that when the number of users in cluster 0 is much larger than the number of independent subbands N (which is usually the case in practical systems), the probability one relay is scheduled on two or more subbands tends to 0. Hence, the objective function of sub-problem 2 can be written as:

$$\underbrace{\begin{aligned} & \max_{\mathbf{A}_0,\mathbf{P}_0} \widetilde{G}_0(\mathbf{A}_0,\mathbf{P}_0|\hat{\mathbf{S}},\hat{\mathbf{H}}) + \sum_{m=1}^{M}\sum_{n=1}^{N}(1-\epsilon)\beta_{0,n} \\ & \max_{\{d_{m,n,k}|\forall k\}} \widetilde{G}^*_m(r_{0,n,m},\{d_{m,n,k}\}|\hat{\mathbf{S}},\hat{\mathbf{H}},\{t_{n,m}=1,t_{n',m}=0\}) \end{aligned}}_{\widetilde{G}^{**}(r_{0\;m}\;)_{,n,m}}$$

It can be proved that $\widetilde{G}^{**}_m(r)$ is a convex piecewise linear function, hence, the Sub-problem 2 become a convex optimization problem and can also be solved by the standard duality approach.

Summary of Solution: The overall decentralized resource allocation algorithm for the relay-assisted CR system is summarized below:

- *Step 1:* For $m = \{0,..,M\}$, mobiles in cluster m deliver the 1-bit RSI to the cluster controller (BS or RS).
- *Step 2:* The mth RS feeds back the function $\widetilde{G}^{**}_m(r)$ to the BS.
- *Step 3:* From the local CSI ($\hat{\mathbf{H}}_0$), RSI and $\widetilde{G}^{**}_m(r)$, the BS determines the power, rate and subband allocation of the mobiles in cluster 0 as well as the RSs using solution of Sub-problem 2.
- *Step 4:* If the mth RS decodes the information from the BS successfully, it will determine the power, rate, subcarrier allocation to the MSs in its cluster based on the local CSI ($\hat{\mathbf{H}}_m$) and RSI using the solution of Sub-problem 1.

The solution is decentralized in the sense that the computational loading is shared between the BS and the RSs. Furthermore, only local CSI is needed at the mth RS and the BS and this substantially reduces the required signaling loading to deliver the GCSI. While the mth RS needs to feedback $\widetilde{G}^{**}_m(r)$ to the BS, the required signaling loading is very small because it turns out that $\widetilde{G}^{**}_m(r)$ is a piecewise-linear function and it can be characterized by O(ML) parameters in the worst case (L is the number of QoS levels).

B. PFS scheduling for RS-Assisted Cognitive Cellular System

The system objective function of PFS is given by $\sum\limits_{m,n,k} \frac{U_{m,n,k}}{\overline{R}_{m,k}}$ where $\overline{R}_{m,k}$ is the average throughput of the kth user in the mth cluster. As a result, the PFS is a special case of the weighted goodput objective considered in the paper. Yet, brute-force applications of the solution in the pervious section in PFS will incur a large signaling overhead from the RS to the BS because the $\widetilde{G}^{**}_m(\cdot)$ of PFS involves very large number of parameters (and hence, induce huge signaling overhead for mth RS to feedback $\widetilde{G}^{**}_m(\cdot)$ to the BS). In the following, we obtain a simple characterization of $\widetilde{G}^{**}_m(\cdot)$ (which is asymptotically optimal) under PFS.

Lemma 1. *Suppose the links between the base station and the relays are sufficiently good, if* K_m *is sufficiently large,* $\widetilde{G}^{**}_m(r)$ *can be simplified as follows in Subproblem 2*

$$\begin{aligned} & \hat{G}^*_m(r) \\ = & \begin{cases} \sum_{n=1}^{N} \frac{r w_{m,A_n}\beta_{m,n}(1-\epsilon)}{4R_m} \log_2(1+p_n l_{m,n,A_n}\varphi_{m,n,A_n}) \\ \sum_{n=1}^{N} \frac{w_{m,A_n}\beta_{m,n}(1-\epsilon)}{4} \log_2(1+p_n l_{m,n,A_n}\varphi_{m,n,A_n}) \end{cases} \end{aligned}$$

where the first equation is when $r \le R_m$ *and the second equation is otherwise;* R_m, $p_{m,n}$ *and* A_n *are the parameters characterize* $\widetilde{G}^{**}_m(r)$, *which are given by* $R_m = \sum_{n=1}^{N} \frac{1}{4}\log_2(1+p_n l_{m,n,A_n}\varphi_{m,n,A_n})$, $\quad p_{m,n} = \frac{\beta_{m,n}P_0}{\sum_{n=1}^{N}\beta_{m,n}}$ *and* $A_n = \arg\max_k w_{m,k}\log_2(1+p_{m,n}l_{m,n,k}\varphi_{m,n,k})$.

Since $\widetilde{G}^{**}_m(r)$ can be parameterized by only one point, the feedback overhead to deliver $\widetilde{G}^{**}_m(r)$ from the mth RS to the BS is very small and does not scale with K_m.

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, we shall compare the performance of the proposed relay-assisted cognitive cellular system with several baseline systems. Baseline 0 refers to a naive design of a cognitive cellular system (without RS) where the power, rate and subcarrier allocation are designed assuming perfect CSIT. Baseline 1 and 2 refer to a similar cognitive cellular system (without RS) except that the control policies are designed for imperfect CSIT. Moreover, in baseline 1, the PU activity in

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[4] $F^{-1}_{|\hat{H}_{m,n,k}|^2/\frac{1}{2}\sigma_e^2}(\cdot)$ denotes the inverse cdf of non-central chi-square random variable with 2 degrees of freedom and non-centrality parameter $|\hat{H}_{m,n,k}|^2/\frac{1}{2}\sigma_e^2$.

[5] Due to the page limitations, we omit some details and all the proofs in this manuscript which are relatively less important. A complete technical report with all proofs can be found in http://www.ece.ust.hk/∼wray/publication.htm.

RS clusters is the same as that in BS cluster, i.e. $q_{pm} = q_{p0}$. In baseline 2, the PU activity in RS clusters is much lower than that in BS cluster, i.e. $q_{pm} = 1 - (1 - q_{p0})^{1/6}$. The overall cell radius of the system is 5000m in which Cluster 0 has radium of 2000m and RS 1-6 are evenly distributed on a circle with radius 3000m as illustrated in Figure 1. MSs randomly distribute in the cell with $K_0 = 10$ MSs in Cluster 0 and $K_m = 5$ in Cluster $m (m = 1, \cdots, 6)$. The path loss model of BS-MS and RS-MS is $128.1 + 37.6 \log_{10}(R)$ dB, and path loss model of BS-RS is $128.1 + 28.8 \log_{10}(R)$ dB (R in km). The standard variance of shadowing effect is 8 dB. There are 64 subcarriers with 4 independent subbands. The average interference constraint to the PU is 0 dB. Each point in the figures is obtained by averaging over 2000 independent fading realizations.

Figure 2 illustrates the PFS objective $\sum_k \log R_k$ and access probability[6] of MSs in Cluster m $(m = 1, \cdots, M)$ versus PU activity q_{p0} at receive $SNR = 10$ dB and $\sigma_e^2 = 0.01$. $\sum_k \log R_k$ and access probability decrease with the increase of PU activities. It can be observed that our proposed scheme provides much greater access probability as well as fairness/throughput performance for the MSs at the cell edge compared with baseline 1 and 2 over a wide range of PU activities. This performance gain is contributed by the conventional RS path-loss gain as well as the increase in the access opportunity for MS at the edge.

Figure 3 illustrates histogram of the average goodput of MSs at various distance from the BS at receive $SNR = 10$ dB and $\sigma_e^2 = 0.01$. It can be observed that baseline 1 and 2 can deliver large system goodput only for those MSs close to the BS. It has very low access probability and average goodput for those far-away mobiles, causing severe fairness issues. However, there is a significant gains in the system goodput of far-away MSs in the proposed system, illustrating both the throughput and fairness advantage over the baseline systems.

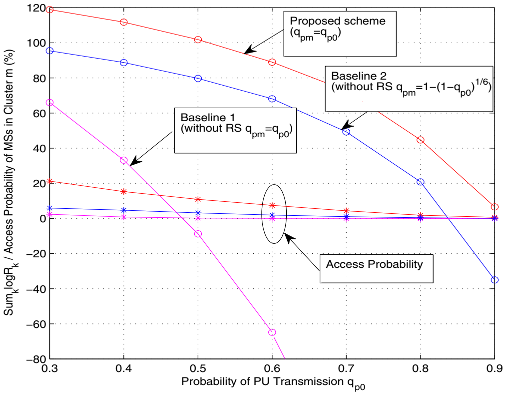

Fig. 2. $\sum_k \log R_k$ and access probability of MSs in Cluster m $(m = 1, \cdots, M)$ versus probability of PU transmission q_{p0} at $q_f = 0.2$, $q_d = 0.8$, M=6, N=4, K_0=10, K_m=5, I=0 dB, receive $SNR = 10$ dB, $\sigma_e^2 = 0.01$, . The number of subcarriers is 64 (with 4 independent multipaths). Baseline 1 and baseline 2 are systems without RS under a PU activity of q_{p0}, $q_{pm} = q_{p0}$ and q_{p0}, $q_{pm} = 1 - (1 - q_{p0})^{1/6}$ respectively.

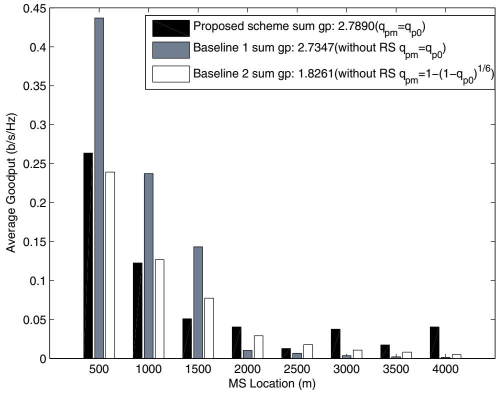

Fig. 3. Histogram of the average goodput of MSs at various distance from the BS at $q_{p0} = 0.3$, $q_f = 0.2$, $q_d = 0.8$, M=6, N=4, K_0=10, K_m=5, I=0 dB, receive $SNR = 10$ dB, $\sigma_e^2 = 0.01$. The number of subcarriers is 64 (with 4 independent multipaths). Baseline 1 and baseline 2 are systems without RS under a PU activity of q_{p0}, $q_{pm} = q_{p0}$ and q_{p0}, $q_{pm} = 1 - (1 - q_{p0})^{1/6}$ respectively.

VI. CONCLUSION

In this paper, we have proposed the design of downlink relay-assisted cognitive cellular system, which has the cluster-based architecture and dynamically shares the spectrum of PU systems. Optimal decentralized algorithm has been derived for joint rate and power control, and subcarrier allocation at the RS and the BS respectively. This algorithm maximizes the weighted system goodput where proportional fair is included as a special case. The algorithmic design has accounted for imperfect channel state information and spectrum sensing measurement, ensuring robust performance. Significant throughput gain has been observed from simulation results.

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[6] Access probability is the probability that a MS on the cell edge access at least one subband in a scheduling slot.